$$\left(\frac{u}{r}\right)_b = \frac{M_1 p_1}{E_2 \beta_1} f_3(r) + \frac{G_1 p_1}{r} \cos\theta$$

(94a-c)

$$\left(\frac{v}{r}\right)_b = \frac{8 M_1 p_1}{E_2 \beta_1} (k_2^2 - 1)\theta - \frac{G_1 p_1}{r} \sin\theta$$

where $f_1(r)$, $f_2(r)$, and $f_3(r)$ are defined by Equations (20a-c) in the text and where

$$\beta_1 \equiv (k_2^2 - 1)^2 - 4k_2^2 (\log k_2)^2 \quad (95)$$

The moment $M = M_1 p_1 r_1^2$ is found by integrating the negative of the Lamé hoop stress $(\sigma_\theta)_c$ for a cylinder given by Equation (13b) in the text over the side of the segment; i.e.,

$$M = -\int_{r_1}^{r_2} (\sigma_\theta)_c \, r \, dr \quad ,$$

hence,

$$M_1 = \frac{-1}{p_1 r_1^2} \int_{r_1}^{r_2} \left\{ \frac{(p_1 - p_2 k_2^2)}{k_2^2 - 1} - \frac{(p_2 - p_1) k_2^2}{k_2^2 - 1} \left(\frac{r_1}{r}\right)^2 \right\} r \, dr$$

$$M_1 = -\frac{1}{2}\left(1 - \frac{p_2}{p_1} k_2^2\right) + \left(\frac{p_2}{p_1} - 1\right) \frac{k_2^2}{k_2^2 - 1} \log k_2 \quad (96)$$

G_1 is found by taking a reference point for the radial deflection u. If the point $r_0 = \frac{r_1 + r_2}{2}$, $\theta = 0$ is fixed,

then

$$G_1 = -\frac{M_1 r_0}{E_2 \beta_1} \left\{ -4(1 + v) k_2^2 \left(\frac{r_1}{r_0}\right)^2 \log k_2 + 4(1-v) \left[k_2^2 \log\left(\frac{r_0}{r_1}\right) - \log \frac{r_0}{r_1} \right] - 4(k_2^2 - 1) \right\} \quad (97)$$

The equations for the total stresses and displacements in ring segments were programmed on the computer and some calculations carried out. Example results are given in Table LV for $k_2 = 2.0$ and $\alpha = 60$ degrees. It is noted that a small residual stress σ_θ remains on the side of the segments. To be more accurate, i.e., to achieve sides entirely free of stress, the residual σ_θ could be removed by using a "dipole" solution in addition to the bending solution. However, the self-equilibrating residual stress that would be removed has a local edge effect according to the principle of St. Venant. Therefore, the σ_θ stresses in Table LVI are believed to be indicative of the actual magnitude of hoop stresses in segments at the center.

TABLE LV. STRESSES AND DEFLECTIONS IN A RING SEGMENT, $k_2 = 2.0$, $\alpha = 60°$, $\nu = 0.3$

r/r_1	σ_r/p_1	σ_θ/p_1	$\frac{Eu}{rp_1}$ at $\theta = 0°$	$\frac{Ev}{rp_1}$ at $\theta = 30°$
1.0	-1.0000	0.0394	0.6324	-0.1301
1.1	-0.9068	0.0123	0.4877	-0.0853
1.2	-0.8310	-0.0033	0.3747	-0.0480
1.3	-0.7676	-0.0112	0.2846	-0.0164
1.4	-0.7137	-0.0137	0.2117	0.0107
1.5	-0.6670	-0.0126	0.1519	0.0341
1.6	-0.6260	-0.0089	0.1022	0.0547
1.7	-0.5896	-0.0033	0.0606	0.0728
1.8	-0.5568	0.0035	0.0254	0.0890
1.9	-0.5271	0.0113	-0.0046	0.1034
2.0	-0.5000	0.0197	-0.0303	0.1163

Appreciable bending, displacement v, is also noted. The bending increases with segment size and angle α as shown in Table LVI. This bending would tend to cause the segments to dig into the liner as shown in Figure 78. Therefore, it is recommended that segments be designed with radii larger than the radii of mating cylinders in order to compensate for the change in radii due to bending. This is illustrated in Figure 78.

Note that the deflection u in Table LV can have an arbitrary translational component; i.e., the segment is free to move radially a constant amount. In calculating interferences, the difference in deflection $u(r_1) - u(r_2)$ at $\theta = 0°$ is used and the constant amount drops out.

ELASTICITY SOLUTION FOR A PIN SEGMENT

A pin segment is shown in Figure 79. Its geometry is defined by the radii r_1 and r_2 and the angle α. r_2 is taken to the inside of the pin holes as indicated. The loading of the pin segment is more complicated than that of the ring segment as shown in Figure 80. A constant pressure p_1 is assumed to act at the inside. A variable pressure is assumed to act at the outside, i.e.,

$$\sigma_r = -p_1, \text{ at } r_1$$

$$\sigma_r = -p_2 (1 + \cos m\theta), \text{ at } r_2 \qquad (98a, b)$$

In addition, a shear acts at r_2:

$$\tau_{r\theta} = -\tau \sin m\theta, \text{ at } r_2 \qquad (98c)$$